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HOW CAN WE MINIMIZE THE EFFECT OF EXAMINATIONS ON SECONDARY EDUCATION?

BY JAMES H. SHIPLEY.

The title implies that the effect of examinations on our secondary teaching is injurious. The author believes that this is true and for the following reasons:

1. Examination set up a false and narrow standard for both teacher and pupil. This is especially true in New York City under the regents' examinations, where pupils are promoted and graduated largely on the results of a three-hour written examination in each subject, and where teachers are advanced in salary largely on the same basis. Under such conditions the whole object of the teaching becomes a burning desire to succeed in examinations, and the teaching degenerates into memorizing and cramming. This is very evidently true in mathematics, and especially geometry, in which a sufficient number of theorems can be memorized to pass the examination; a person high in authority said that if too many original problems were given, for the purpose of testing the pupils' power to reason, a sufficient number of pupils would not pass the examinations to make them worth while. This is a confession that the examinations must be more or less mechanical; the teaching of course must correspond.

This false standard of teaching set up by examinations has been widely condemned: Mr. Horner, chief of the Examination

Division of the N. Y. State Board of Regents, says: "Ofttimes the examinations operate to emphasize stuffing at the expense of leading. Teachers sometimes subvert the whole purpose of the schools by teaching with the syllabus in one hand and old question papers in the other, and lead students to believe that the main business of the schools is the passing of examinations." In fact, this is the general policy.

Dr. Lyttle, who wrote the history syllabus, says: "I have believed the regents' examinations to be of immeasurable value in establishing and maintaining fair educational standards, but . . . both pupils and teachers too often come to consider examinations not as one of the means, but as the end of education."

Hon. Mr. Downing, First Assistant Commissioner of Education, State of New York, says: "The bane of all this matter of education is the measuring of attainments in counts and points; there is no question of the evil effects of it in college or in secondary schools."

President Hadley, of Yale: "Proficiency in subjects studied during the few months previous to the examination is at best largely a matter of memory; and it not infrequently happens that such memory is most highly developed in those pupils who have done comparatively little real thinking for themselves. This difficulty may be lessened by skill in arranging the examinations but can never be wholly eliminated. The written examination even in the best hands is apt to be a test of the range of a students' proficiency rather than of its thoroughness."

Hon. Henry Sabin, Des Moines, Iowa: "One half the work in our schools is ruined for education where the fetich of examinations is held up prominently before the pupils as the object of study. The day the pupils begin to study for marks, they conceive a false ideal of success, to attain which occupies all their thoughts. When the child comes to feel that 80 per cent. is success and 79 per cent. is failure, it would be better for him that he had never seen the inside of the schoolroom."

Professor Seeley, Department of Pedagogy, State Normal School, Trenton: "Pernicious indeed does such a system become when the Board of Education and the community estimate the efficiency of the teacher and of the school by the proportion of pupils that pass the examinations. Add to this the practice

of advancing the teachers themselves upon the basis of their success in getting pupils through examinations, and the worst evils of such a system are realized."

Editorial, *School and Home*, Atlanta, Ga.: "Examinations are means, pure and simple, and should never be ends. The student should never be allowed to conclude from our instruction that he is studying for an examination."

A certain New York City high-school teacher, who had a long series of A's to her credit for supposedly excellent teaching, once said at a teachers' meeting: "I am receiving the highest ratings for my teaching, but I know that I am not teaching right; I have been told to get the pupils through the examinations and that is what I am doing."

Professor Jacobs, of Brown University, says: "Examinations are the children of degree-giving; in the high schools, examinations are the children of the diploma. They are the most efficient aids in making the machine system of the school felt. Given in regular doses, examinations can make a very poor teacher out of a very good one in a very short time."

Chas. F. Wheelock, Assistant Commissioner for Secondary Education, State of New York, recognizing the harmful effect of written examinations on the quality of the teaching, says: "May we not be looking forward to the time when we can dispense with some of our examinations, when we may differentiate between the subjects in which examinations are profitable and those in which they are unprofitable." They are unprofitable in all subjects.

Miss Johnston, a New York State District Superintendent, said that when she was teaching, regents' examinations helped her in her class-room work in physics because she did not know how to teach it, but hindered her in teaching English, which she liked and did know how to teach.

Boston does not allow examinations in English because it hampers the teaching.

The High School of Commerce, New York City, does not give regents' examinations because they narrow the presentation of the work.

Mr. Horner, Director of Department of Examination and Inspection, New York State Educational Department, says: "Do

not our examinations, for better or for worse, determine to a very large extent what we shall teach and how we shall teach it? Do they not have an appreciable controlling influence upon the organization of our schools, upon the promotion of our pupils, and even upon the selection and promotion of our teachers? We must unhesitatingly, it seems to me, answer these questions in the affirmative."

3. Another serious objection to long, final, written examinations is the enormous waste of time entailed in giving them. In New York, the regents' examinations are given about the middle of January and June; this practically marks the end of the term, as the pupils know that the term's work has been finished and it is almost impossible to induce them to do any serious work; many leave in June as soon as the examinations are finished. This means about four weeks out of the year lost to class-room instruction; adding to this the four weeks preceding the two examinations usually given over to intensive cramming (called by the regents useful reviewing), and we see that the examinations use up about 20 per cent. of the total school year.

On this point Professor Seeley says: "The examinations require several days, during which the regular exercises are abandoned and the school thrown into confusion; the whole machinery of the school is disorganized, an unnatural and disquieting spirit pervades the pupils, the nervous tension is very great, and the best results of school-work cannot be secured."

Thomas W. Churchill, formerly President New York City Board of Education, says: "The Hanus report and the newspapers remark the waste of pupils' and teachers' time by an elaborate system of regents' examinations thrust into the high-school programme twice a year, disorganizing the orderly progress of studies, removing from our own system to a central office at Albany decision upon such questions of promotion and graduation as should properly depend upon our own teachers. Too many educational authorities condemn the outside examination to warrant this board in following the lead of its predecessors who abandoned home rule in the management of their high schools for the avowed purposed of securing a subvention from the state. I recommend a thorough inquiry into the worth of the regents' examinations."

4. The regents' examination system as applied in New York City is objectionable from the standpoint of economy, both in money and energy. During the two weeks that regents' examinations are held, more than 1,500 high-school teachers are detached from their regular business of teaching and paid as high as $60 a week for acting as monitors in examination rooms, and correcting papers, which are again marked at Albany at an added expense to the State for the same work, and some papers are even gone over a third time by the board of superintendents. It costs New York City about $100,000 to conduct examinations for two weeks each year, and an almost equal amount for reduced efficiency in teaching during the confusion following the examinations, and another similar amount diverted from paying teachers for real teaching and applied to paying them for stuffing the pupils for two weeks preceding each examination, not to speak of a general inefficiency all through the term due to misdirected energy.

5. The worst feature of long, final examinations is their injustice to the pupils and their injurious effect on the health of the pupils. The schools are supposed to instill a sense of justice in the pupils, but here they have constantly before them an almost perfect example of injustice and also inefficiency. One of the brightest, quickest and most conscientious girls in a certain high school received 60 per cent. instead of 65 per cent. in a regents' algebra examination and was thereby kept out of training school and remanded to the high school for another whole term; her failure was due to her taking five examinations in one day, not even stopping for lunch. She said, weeping: "This world is ruled by injustice." This is an illustrative case of hundreds that occur in every examination. Good pupils frequently fail and mediocre ones pass. Nichols says: "It is hard to tell which has the worse influence upon school work—the failure of a good pupil or the success of a poor one." The conscientious work of a whole term or year is set at naught by mere chance; one term, 90 per cent. of the pupils will pass the examination, and the next term only 40 per cent.; a pupil (as the one mentioned above) will get 60 per cent. in one examination and 90 per cent. in the next, and not know any more about the subject. A boy, on being asked how he was doing in Eng-

lish, said: "I am not doing much, but I'll be all right on the exam."

Again, the results of the re-reading at Albany are so late in coming back to the schools that the pupils are well advanced into the work of the next term when the revised markings make it necessary for them to change their programs, too late for effective work. In one year, the regents rejected 20,000 papers that were claimed as passing by the schools.

On the point of injustice to the pupils, Professor Seeley says: "When a pupil who falls but a fraction of one per cent. below the fixed passing mark is required to go over the whole year's work the matter becomes of vital importance to both pupil and parent. This system has a tendency to drive pupils from school because they cannot bear the thought of spending another year upon subjects that they have nearly mastered, or in which they have failed through a possible accident in an examination.

Superintendent Gilbert says: "The promotion of no child should be based upon examinations; his time in school is limited, and to keep him back a half year or a year for any reason other than his own personal good is to commit a serious offense against him. The system has its claims, but promotion is an individual matter, and a single child whose case is under consideration is entitled to a judgment according to his needs, not according to the needs of his neighbors or of the system as a whole. The system that cannot stand the shock of proper consideration for individuals is an unworthy system."

As to the effect on the pupils' health anyone can determine it intuitively. Dr. Sutton, of the University of Texas, says: "Testimony to the effect that the health of many pupils has been injured because of extraordinary labors in preparing for an examination is abundant." Professor Chancellor: "Common experience as well as medical science assures us that the long written examination is a crime against the health of most girls and many boys." Cooper (in "Educational Foundations") says: "Why does this fell moment of examination day hang like the sword of Damocles above the otherwise care-free student? Who invented it? Who can abolish it? Such a benefactor would be placed in the pantheon of schoolboy immortality and worshipped age without end by the youth of the world. For

pupils and instructors alike, examinations seem to have become the bêtenoir of education, the cynosure of all educational hopes and fears."

The *School Bulletin* of Syracuse said that a recent "unpardonable test" in algebra (in which only about 40 per cent. of the city's pupils passed) seemed to come from the asylum for the insane. "Think of the worry," it says, "of the 30,000 pupils over these problems, the tears that were shed, the nights that were sleepless, the wear and tear! Make the recurrence of such a crime impossible."

Assuming that any kind of supervision over the schools should be for the purpose of improving the teaching and not for the purpose of rating teachers in order to determine their salaries, any fair-minded person must admit that the only way to be able to suggest improvements in the teaching is to first see the teaching. It sounds like good logic to say that a good teacher ought to be able to get his pupils through the examinations, and the more he gets through, the better the teacher he is, and that therefore the "goodness" of a teacher can be determined by the percentage of his pupils that pass the final examinations. This of course is true if we accept the passing of examinations as the object of education, but it is false if we assume that the object of education is to teach the pupils how to use their minds; in fact, up to a certain limit, the percentages of pupils passed in the examinations and the real worth of the teachers who pass them are *inversely* proportional, because to force pupils through the examinations the teacher must resort to intensive drill and other devices that do not constitute real teaching. In a class of sub-normal pupils, anemics, and others, the teacher may be doing exceptionally good work, but the results of an examination as showing her value would be deplorable; the same is true to a diminished extent in the high school—a teacher may be doing his best work with a class that shows up the poorest in the examinations, simply because the material is poor. In short, the only way to form any idea of the ability of a teacher is to watch him teach; and the only reasons for watching him teach are to tell him how to teach better (if necessary), or leave him alone (if he is teaching well enough), or carry his methods to other teachers (if he is doing especially well).

This could be done by close class-room inspection, and would have to be done if examinations, now used as the criterion for everything, were abolished.

The regents recognize the usefulness of inspections. Commissioner Horner says, in regard to abolishing examinations in the elementary schools (and the same argument holds for the high schools): "No amount of examining, no amount of rating papers, no amount of reporting on this matter or that matter, and no amount of academic discussion will take the place of the helpful suggestions and the inspiring influence which district superintendents may be personally able to bring to the schoolrooms under their supervision." Mr. Horner says: "The purpose of examinations is only half accomplished after the examinations have been given and the papers rated by the State Educational Department. The even more important work then begins when the inspections force takes up the records of examinations and goes back to the schools with personal suggestions upon how to correct evils which may exist." That is to say, it is necessary to upset the regular work of every high school in the state for a period of one month each year, and at a cost in diverted salaries of about $300,000, in order to find out where evils exist so that the inspections department can get after them! And they are not after the evils only; Mr. Horner further says: "It is our express purpose to study the results of examinations and through visitations to trace those results, be they good or bad, to their sources in the schools." This seems to mean that *all* schools will be visited by the state inspectors, so why not use this visit for the purpose of finding out what evils exist as well as doing all that is possible to correct them? Surely this work can be done a hundred times more effectively through actual inspection of the school and the class-room instruction than by a system of examinations miles removed from the affected school. "Last year," Mr. Horner writes, "we inspected every public secondary school in the state outside of the cities and villages with a view to ascertaining what the particular problems are in these schools to which we should address our attention." Of what possible help could examinations be in this work? Commissioner Horner says further: "It is not possible for this department to inspect the schools as it would wish to do, with ten

inspectors to visit upwards of 1,000 secondary schools." Then why not appoint more; enough money could be saved from the discontinuance of examinations to employ all the inspectors needed.

Commissioner Wheelock says: "It is not possible to visit even in a perfunctory way, each year, each of the institutions of the university, nor is that necessary; it may be assumed that most of the city high schools are under expert local supervision and therefore in such schools state inspection is less urgently demanded, although for many reasons still desirable." State Commissioner of Education John H. Finley says: "It is of the utmost importance, therefore, that the university maintain a force of trained specialists in each group of subjects so as to be able to carry to individual schools the cumulative experience of trained teachers the country over and the collective wisdom which a large central office, instituted upon a professional basis, must possess if it is to justify its existence. Such a faculty will take large note of the existence and the administration of our examination system and of its desirable and necessary continuance; but will at the same time hold clearly before the teachers of the entire state the patent fact, which many of them at times strangely overlook, that the primary business of the schools is *teaching* and not *examining*." It is not "strange" at all that teachers do this; it is the natural impulse of human nature—the examinations being there to pass, and the teachers' advancement depending upon the passing, he is going to concentrate all his energies toward that end; to advise him to do otherwise is like forcing a man to walk barefooted on a skating pond and at the same time urging him not to get cold feet.

7. Examinations should be abolished for the reason that anything that is now determined by examinations can be determined more efficiently some other way: for example, as to the promotion and graduation of pupils by examinations, Mr. Deffenbaugh, of the United States Bureau of Education, says: "Few schools now rely entirely upon examinations as a test for promotions. In some schools examinations are made to count one half, while in others they count only one fourth. It is now recognized that examinations are not a true test of the pupil's ability to do the next grade's work, and that a pupil should be

promoted not on what he knows but what he can do. Of 756 superintendents in the smaller cities reporting, 669 say that they are now depending on examinations much less than formerly." Wm. Chancellor: "In the war which all educators and other competent persons are waging against written examinations either as finalities or as the deciding factors in promotion. . . . This systematic attack upon the written examination which is going on everywhere in the land is not to stop until the examination ceases to exist." Mr. Sabin (quoted above) says about promotions: "The system which makes the promotion of children from grade to grade during any part of their school life dependent upon a certain per cent. as determined by a written examination is faulty in its construction and injurious in its results." Professor Gilbert (above quoted) says: "The promotion of no child should be based upon examinations. The examination fetich still has its worshippers, but in so far as I know there are now comparatively few school systems in which reliance is placed wholly upon examinations in determining the promotion of pupils."

Dr. Sutton, of the University of Texas: "The formal examinations conducted at the close of the school term has been employed for centuries as a means of determining the efficiency of instruction; but in the educational world it has long been debated whether this form of testing pupils should be regarded as pedagogically sound. If justice is to be done the individual pupil, other than mere knowledge-tests should be employed."

The first question asked by the superintendents is, "What are you going to substitute for examinations if you abolish them." The answer is very simple: "Short written tests in the class as a part of the regular work, the promotion of the pupil being based on these tests and the teacher's estimate of the pupil's general ability and his work in class." This is the method used in the Newark High Schools; at Cornell the tendency is away from the long final examinations, with more stress on frequent quizzes; at Amherst college a short test is given each day in each subject; in the New York City High School of Commerce the pupils are promoted on the teacher's estimate based at option on a mid-term examination. In order not to let too much depend upon the teacher's off-hand estimate, there should

be a specified amount of time given to these short written tests, say forty-five minutes per week, given in two, three or four short tests or one long one, subject to inspection by the principal or superintendent.

There are many advantages in this plan: it examines the pupil on practicaly the whole subject and on anything that the teacher wishes to emphasize; it forces the pupil to follow the work understandingly from day to day, which is more beneficial than a furious spurt at the finish; it fosters closer attention in the class because there can be no bluffing or dodging or postponing of work, and any neglect is *at once* reflected in the tests; it gives a continuous incentive to do honest individual work, for otherwise the frequent tests cannot be passed; it keeps before the pupils an object lesson in daily industry and not one of procrastination, indifference, and injustice as under the present system; it is absolutely just to the pupil, judging his ability by twenty-five or fifty tests instead of one; it keeps the teacher in close touch with the pupil, and gives the pupil a continuous estimate of himself, so that he can each week or day make up for his shortcomings and be in a "passable" condition at the end of the term, and not, as now, have to wait passively and usually contentedly until the end of the term (and sometimes several weeks after the end of the term) for the state to tell him whether or not he is a good pupil; the system of frequent short tests allows the regular class-room instruction to be continued right up to the last day of the term without loss of time, money, or energy in a long gruelling series of examinations. Professor Seeley says: "Even with adult students the test will often fully satisfy all the demands and fulfill the requirements which the examination is expected to meet." Dr. Sutton (University of Texas) says: "It would be far better in the opinion of many teachers if the child's eagerness to master his studies should be distributed over all the weeks of the term and not be concentrated on the closing weeks."

Another purpose served by the examinations according to the authorities is that they help to systematize and clarify the term's work. The *review* of the work in *preparation* for the examination may do these things, but it could be done a hundred times more effectively and beneficially by several reviews at the right

time during the term, and for the right purpose, rather than for the present undeniable purpose of cramming pupils for the examinations.

Another claim for the examinations is that they constitute an essential part of a child's education and give him a necessary training of a peculiar kind that he cannot otherwise get. Mr. Fitch, of the Board of Regents, eloquently says: "Pupils must have the spur of ambition, the joy of conflict, the hope of reward, the dread of failure. Examinations are concordant with the universal law of life that struggle precedes success, is necessary to it. Competition rules the court, the cabinet, the camp, the mart. Life is exertion throughout, and examinations do for the younger what its sterner conflicts do for those of maturer years." Eloquence is about all there is to this exposition; as to the "spur to ambition" there likely would be more of it if a pupil knew (as he would in the test plan) that after working faithfully and well he would pass at the end of the term. "The joy of conflict," incident to examinations, has never been very noticeable—in the rare cases where any joy has been detected there was no conflict, and with the mass of pupils where the conflict was patently present the joy seemed to be lacking. "The hope of reward" is well worded; in the regents' system of examinations reward is never *assured* for good work—it is at best only a "hope"; in the plan of frequent short tests, commensurate reward *is* assured—good work is rewarding by passing, and poor work by failing; the pupil knows this and has it continually before his eyes; he is daily shown the invaluable lesson of consequences for his actions. As to the "dread of failure" quoted above, no child should ever be led to "dread" anything that he is supposed to be able to do—he ought to be taught to go at it with confidence or resignation. Dr. Sutton (quoted before) says that in examinations "pupils may manifest some courage, but it is the courage of desperation. Courage and self-reliance are features which should be cultivated regularly, and are not best promoted by spasmodic efforts separated at long intervals." Professor Chancellor has shown scientifically (and it is well known) that there is no fixed correlation between success in examinations and success in life. Professor Seeley (quoted before) says: "It is a splendid thing to possess

the power of gathering one's self together for a supreme effort to do an unusual amount of work—such exigencies exist all through life; there is no objection to calling upon pupils to perform unusual tasks occasionally, but they should be prepared for such occasions by regular, steady instruction. Examinations lay stress upon what should be a mere incident in a scheme of education."

Mr. Horner, Chief of the Examination Division, New York State Board of Regents, says that "the secondary-school examinations exist in this state mainly for two fundamental purposes: (1) To test the knowledge and capacity of students and to provide a basis for the issuance to them of useful credentials giving evidence of that knowledge and capacity. (2) To test the quality of the teaching in the schools and to set up in the schools throughout the state safe minimum standards of efficiency and productivity." As for the first of these purposes, it is generally admitted that examinations are not a fair test even of the pupil's "knowledge"; they may test his "capacity"—that is, the amount that he can hold until after examination; the best place to test the pupil's ability in any respect is right in the class-room, and the best way is by the test method described above. It is outside of the function of the state to test the pupils at all—that should be done by the schools; the state's duty has been done when it sends its inspectors into the schools to see that they have adequate courses and equipment, and good teaching. Once a school has been approved by actual "visit and search," any pupil completing the course in such school should be awarded an academic diploma or a qualifying certificate to enter the professions, or a college entrance diploma; and expensive, time-wasting, unjust examination is not necessary as a "basis for the issuance of credentials"—the inspectors should simply say whether or not a certain school is a good school, and then trust to the school to do the rest. This is already done in the case of the qualifying certificate, so why not for graduation also? It is not necessary for the State Educational Department to hold such a strangling grip on the schools—all that it should be concerned with is to find out if every community in the state has good schooling; the best way to find that out is to send an inspector, not to give an examination. The present

method of granting state money to schools should be reversed—instead of giving it to approved schools, it should be given to localities where the schools are poor, so that they can be made good.

As for the second purpose of examinations, as advanced by Mr. Horner, the only way to "test the quality of the teaching in the schools" is to see it; and as for the examinations "setting up safe minimum standards of efficiency and productivity" there is nothing to it—in their working and in their effect on the work in the schools, the examinations are one of the finest examples of inefficiency in the state.

One other reason advanced for not doing away with regents' examinations is that state scholarships are awarded on the results of the examinations; this is another of the many things in the educational world that are done backwards—pupils that pass the examinations with the highest percentages are not always those that the state should select to train at its expense. The pupils most entitled to state help and at the same time best suited to it can best be selected right in the school, where all the attendant factors are known.

To sum up, the present wasteful examination system in New York State should be abolished, and all the purposes that it is supposed to serve should be attained in the following ways:

1. Inspectors to personally visit the high schools to see whether or not the subjects required by the state are being taught, and to see that good teaching exists.
2. Pupils to be promoted and graduated on teachers' estimates based on a certain amount of written test work.
3. Graduates of schools, approved by the regents, to be granted all desired state credentials.
4. State money to be distributed where most needed.
5. State scholarships to be granted to pupils recommended by the schools.

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GRAPHS OF EXPLICIT FUNCTIONS.*

By Robert E. Bruce.

Graphs of Explicit Functions.

In elementary mathematics the explicit function plays a very important part. In spite of this fact the special methods of curve-tracing discussed in English text-books† are adapted more particularly to implicit functions. These methods may of course also be used with the explicit; but if one is interested mainly in the latter simpler methods are available.

As familiar illustrations of the use of the graphs of explicit functions in elementary work, two entirely dissimilar sets of problems may be mentioned the study of which is to some extent simplified by the use of methods described in this paper. The first of these sets is composed of the three famous problems of the Greeks. The cissoid, conchoid, and other curves used in the duplication of the cube, the trisection of the angle, and the squaring of the circle may be expressed as explicit functions and their graphs constructed without great difficulty. The second is the problem of solving for real roots rational integral equations of high degree and transcendental equations. The common graphic method of dealing with the transcendental equation, by breaking it up into two simultaneous equations, may of course also be applied to algebraic forms; and in either case the methods of construction here discussed may frequently be found useful in solving the problem. As an example the solution of

$$\arcsin x + \frac{1}{2^{x+1}} = 3$$

is given in Fig. 3. For reasons that are explained later the figure must be held upside down and the graph referred to O_{IV}

* Presented at the spring meeting, 1917, of the Association of Teachers of Mathematics in New England.

† For example, Frost or Johnson.

as origin. The equation has been broken up into the set

$$\begin{cases} y = \arcsin x, \\ y = 3 - \dfrac{1}{2^{x+1}}. \end{cases}$$

The x of the intersection is about $\frac{1}{2}$, which on substitution gives

$$\frac{5\pi}{6} + \frac{1}{\sqrt{8}} = 2.97.$$

It is not the intention of the writer to give the impression that the methods here discussed are new. They are, as a matter of fact, used to a limited extent in some of our textbooks; but so far as the writer is aware no adequate discussion of them is given. In addition let it be said that there is no thought of suggesting that these methods be substituted for those now in use. Their office is rather to supplement present methods; and when so used they will frequently enable the student to draw with comparative ease graphs that were practically beyond his power so long as he was limited to the elementary method of plotting points from a table of computed values of x and y.

In this elementary method the first step is algebraic. That is to say, values of y are built up from values of x by algebraic processes—the particular processes involved depending on the form of the function to be plotted. In the methods here discussed we substitute largely for this algebraic step the graphic step of building up the graph of the function in question from simpler graphs—the particular graphs used in any case depending upon the form of the function involved. It will be assumed then from the start that the student is familiar with certain simple graphs, just as he is familiar with the algebraic processes by means of which he finds the value of y from the value of x. The list given below contains all the forms that would ordinarily be used and is distinctly longer than the student needs at the start. Much can be accomplished with but a few forms.

List of Elementary Graphs.

1. $y = mx + b$.
2. $y = ax^2 + bx + c$.

3. $y = x^n$ for $n = 1, 2, 3, -1, -2$.

4. $y = n^x$ for $n = 2, e, 3$.

5. $y = \log_n x$ for $n = 10$ and e.

6. The trigonometric functions.

7. The arc functions.

As already indicated it is the purpose of the paper to discuss graphic methods for deriving from these simple forms the graphs of others that are more complicated. With this purpose in view we attack the following question the answer to which, as will be seen, supplies the methods demanded:

Given the graphs of $y = f(x)$ and $y = \phi(x)$ what relationships do the graphs of the following bear to them?*

1. $y = f(x) \pm \varphi(x)$:

$$y = f(x) \pm c,$$

$$y = f(x \pm c).$$

2. $y = f(x) \cdot \varphi(x)$:

$$y = cf(x), \quad y = -f(x),$$

$$y = f(cx), \quad y = f(-x),$$

3. $y = \dfrac{f(x)}{\varphi(x)} = f(x) \cdot \dfrac{1}{\varphi(x)}$:

$$y = \frac{1}{f(x)},$$

$$y = f\left(\frac{1}{x}\right).$$

4. $y = \sqrt{f(x)}$.

5. $x = f(y)$.

This list, it will be observed, is by no means exhaustive. In particular, $y = \sqrt{f(x)}$ is but one case under the more general form $y = [f(x)]^n$. The list is, however, sufficiently exhaustive for all ordinary purposes. No attempt will be made to study

* It will be observed that the use of the function notation, while convenient, is not absolutely necessary.

all the cases in detail. It is believed that no difficulty will be experienced in supplying the reasoning where it is omitted. As one would expect, the effective use of the methods depends to a large extent on the use of co-ordinate paper. Unfortunately it has not proved practicable to make this appear in the figures.

I. Given the graphs of $y=f(x)$ and $y=\phi(x)$ to find the graph of $y=f(x)+\phi(x)\equiv F(x)$. (See Fig. 1.)

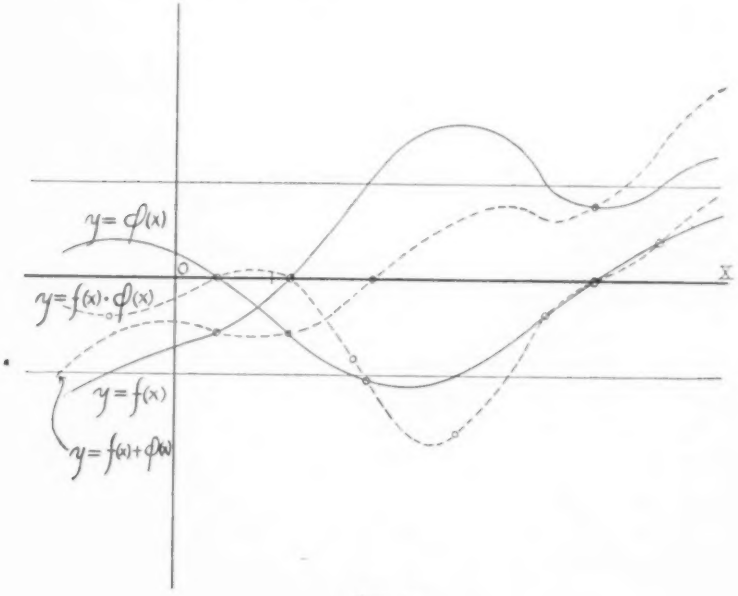


FIG 1

Since the ordinates of $y=F(x)$ are the sums of the corresponding ordinates of the two given curves, it is clear that when either of the given curves crosses the x-axis, $y=F(x)$ crosses the other. $y=F(x)$ crosses the x-axis when the given curves are equally distant from the axis but on opposite sides of it. When the given graphs are on the same side of the x-axis, the graph of $y=F(x)$ is on the same side but at a greater distance from the axis. When the given curves are on opposite sides of the axis, the graph of $y=F(x)$ is between them. In addition to the points located at once from the considerations given above, as many more as desired may be located by adding

ordinates of one of the given curves to the corresponding ordinates of the other with the dividers.

II. Given the graphs of $y=f(x)$ and $y=\phi(x)$ to find the graph of $y=f(x)-\phi(x)\equiv F(x)$. (See Fig. 2.)

In this case the particular illustration $y=2^x-x^3$ is given. Obviously, since the ordinates of $y=F(x)$ are the algebraic differences of the corresponding ordinates of the given curves, when the curves intersect $y=F(x)$ crosses the x-axis. The effects of the change of sign of x^3 and of the relative distances of the two curves from the x-axis are readily seen from the figure.

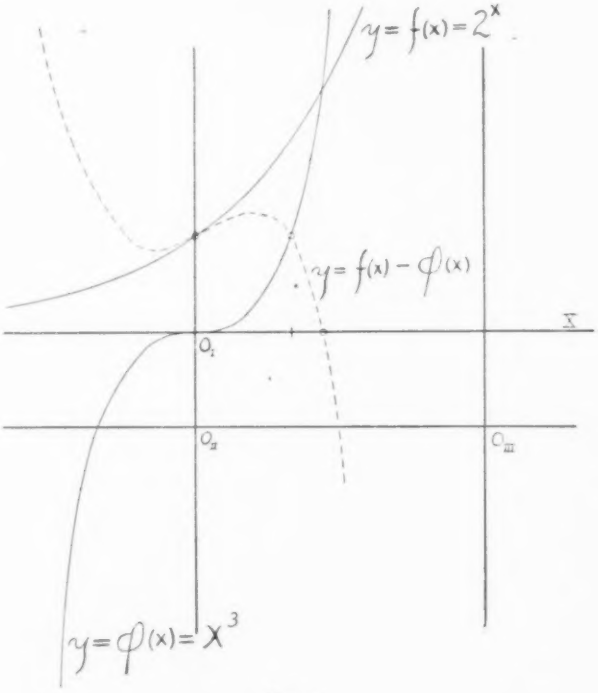


FIG. 2.

III. Given the graph of $y=f(x)$ to find the graphs of $y=f(x)\pm c$ and $y=f(x\pm c)$.

The first of these forms is clearly a special case of $y=f(x)\pm\phi(x)$ where $\phi(x)$ is a constant. The result in the

special case may however be easily determined independently of the general discussion. Obviously the ordinates of $y = f(x) \pm c$ differ from those of $y = f(x)$ by the constant value c. Hence to draw the graph of $y = f(x) \pm c$ it is sufficient simply to draw the graph of $y = f(x)$ and then to lower or raise the x-axis the distance c according as the graph desired is $y = f(x) + c$ or $y = f(x) - c$. For example, in Fig. 2 if the graph is referred to O_{II} as an origin it becomes the graph of the function $y = 2^x - x^3 + 1$. This transformation of the x-axis makes it possible in drawing the graph of a function to disregard an absolute term until the plotting is complete.

A corresponding simple transformation of the y-axis makes it possible to determine the graph of $y = f(x \pm c)$ directly from the graph of $y = f(x)$ as follows. If a is substituted for x in $f(x)$ the resulting ordinate is $f(a)$. The value of x that gives the same ordinate in $f(x + c)$ is obviously $a - c$. For the substitution of $a - c$ for x in $f(x + c)$ gives $f(a)$. Hence for every point on $f(x)$ there is a point having an equal ordinate on $f(x + c)$, this latter point being c units to the left of the corresponding point on $f(x)$. Hence to find the graph of $f(x + c)$ we plot the graph of $f(x)$ and then move the y-axis c units to the right. For example, in Fig. 2 if the graph is referred to O_{III} as origin it becomes the graph of

$$y = 2^{x+3} - (x + 3)^3 + 1 = 2^{x+3} - x^3 - 9x^2 - 27x - 26.$$

Similarly the graph of $y = f(x - c)$ may be derived from the graph of $y = f(x)$ by moving the y-axis c units to the left. As will be seen later these transformations are particularly useful in plotting the graph of the irreducible rational integral function.

IV. Given the graphs of $y = f(x)$ and $y = \phi(x)$ to find the graph of $y = f(x) \cdot \phi(x) \equiv F(x)$. (See Fig. 1.)

Since for any value of x the ordinate of $y = F(x)$ is the product of the corresponding ordinates of $y = f(x)$ and $y = \phi(x)$, when either of the latter graphs crosses the x-axis, so also does the graph of $y = F(x)$. Moreover when either of the given graphs crosses the line $y = 1$, $y = F(x)$ crosses the other. If the given graphs are on the same side of the x-axis the required graph is above the axis, otherwise it is below the axis. The position of the required graph depending on the relation-

ships between the given graphs and the lines $y = \pm 1$ are easily determined.

V. Given the graph of $y = f(x)$ to find the graphs of the following:

$$y = cf(x), \qquad y = -f(x),$$
$$y = f(cx), \qquad y = f(-x).$$

Since the ordinates of $y = cf(x)$ are c times those of $y = f(x)$ the following device may be used for deriving the graph of the former from that of the latter. When c is positive plot $y = f(x)$ using on the y-axis a unit c times as great as that used on the x-axis; then interpret the graph as though the unit on the y-axis equalled that on the x-axis and the result is the graph of $y = cf(x)$. If c is negative it is simply necessary to plot $y = |c| \cdot f(x)$ in the way outlined above but with positive ordinates plotted downward and negative ordinates upward. In particular to discover the graph of $y = -f(x)$ it is sufficient simply to plot $y = f(x)$ and then, rotating the paper 180 degrees on the x-axis, to examine the graph *through* the paper by holding it toward the light. $y = f(x)$ and $y = -f(x)$ are symmetrical to each other with respect to the x-axis.

By a process of reasoning similar to that used above it is clear that $y = f(cx)$ may be derived from $y = f(x)$ by changing the unit, or direction, or both on the x-axis before plotting. In particular the graphs of $y = f(x)$ and $y = f(-x)$ are symmetrical to each other with respect to the y-axis and either may be derived from the other by a rotation of 180 degrees about that axis.

It is clear then that to derive $y = -f(-x)$ from $y = f(x)$ it is simply necessary to turn the graph upside down.

Before proceeding farther it may be pointed out that in all but the simplest cases it will be found useful before plotting to arrange a scheme of the work. In making the scheme one naturally starts with the required form and proceeds step by step to simplify it. The actual work of plotting proceeds in the opposite direction. For example, the scheme given below was built from the bottom upward but the plotting was done in the reverse order. As in the case of the other illustrations in the paper, a simple example is used. It is clear however that the

methods may be applied to forms that are much more difficult. The graph is given in Fig. 3.

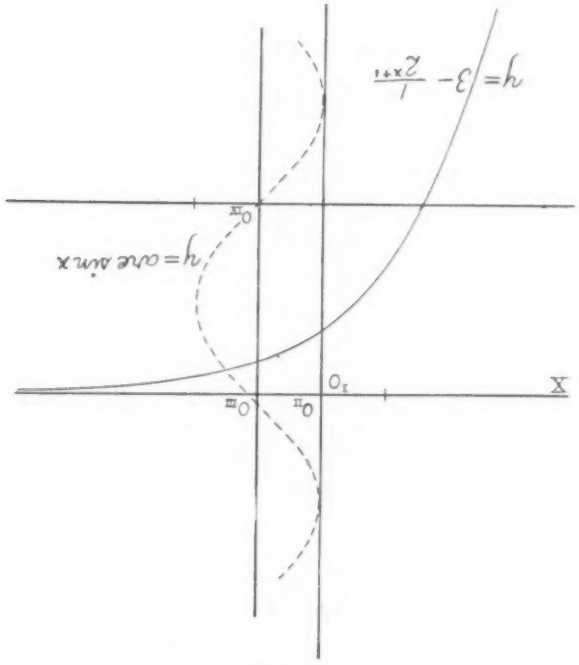


FIG. 3.

SCHEME OF WORK FOR PLOTTING $y = 3 - \frac{1}{2^{x+1}}$.

Step.	Result.
Plot $y = 2^x$	$y = 2^x$
Turn paper upside down	$y = -\frac{1}{2^x}$
Move y-axis one unit to the right	$y = -\frac{1}{2^{x+1}}$
Move x-axis three units down	$y = 3 - \frac{1}{2^{x+1}}$

VI. Given the graphs of $y = f(x)$ and $y = \phi(x)$ to find the graph of

$$y = \frac{f(x)}{\varphi(x)}.$$

Perhaps the simplest method of attacking this problem is to change the quotient into the product of the dividend and the reciprocal of the divisor. The work may then be completed as in IV above. We therefore consider the following.

VII. Given the graph of $y=f(x)$ to find the graph of

$$y = \frac{1}{f(x)}.$$

(See Fig. 4.)

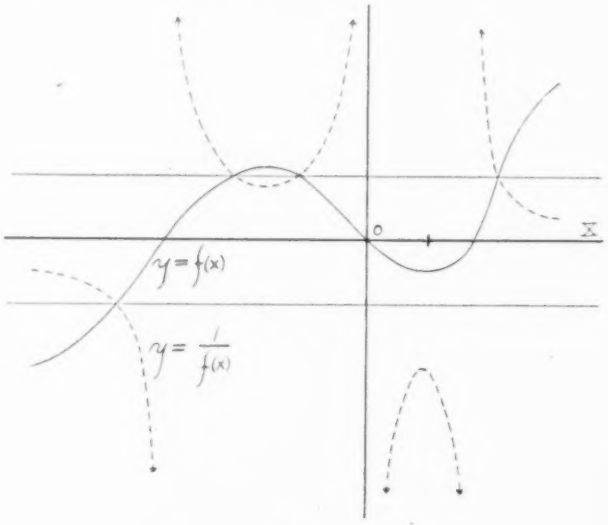


FIG. 4.

As the ordinates of the graph of the required function are the reciprocals of the corresponding ordinates of the given function, it is clear that whenever the graph of $y=f(x)$ crosses the x-axis the graph of

$$y = \frac{1}{f(x)}$$

goes to infinity and vice versa. The signs of the corresponding ordinates of the two graphs are the same. They intersect on the lines $y=\pm 1$. Points between these two lines on either graph correspond to points outside on the other.

VIII. Given the graph of $y=f(x)$ to find the graph of

$$y=f\left(\frac{1}{x}\right).$$

(See Fig. 5.)

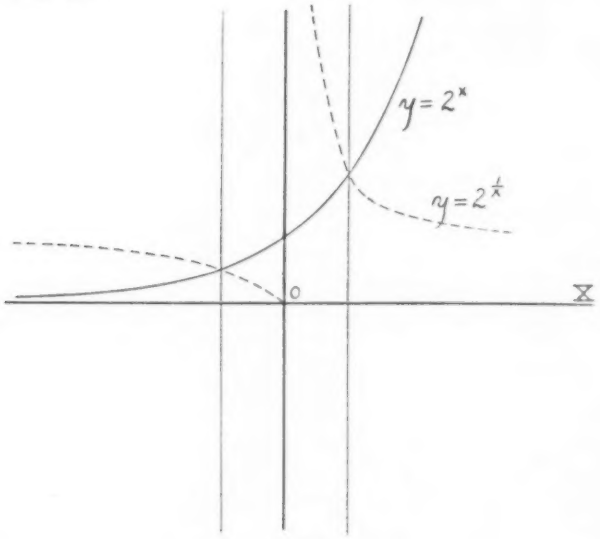


FIG. 5.

For every ordinate which the graph of $y=f(x)$ has between the lines $x=\pm 1$, the graph of

$$y=f\left(\frac{1}{x}\right)$$

has an equal corresponding ordinate outside the lines and vice versa. For if $f(x)$ becomes $f(a)$ when $x=a$, $f(1/x)$ becomes $f(a)$ when $x=1/a$. Speaking roughly, the parts of the graph of $y=f(x)$ exterior to the two lines are reversed in direction and crowded within the lines in the graph of $y=f(1/x)$. Similarly the parts of the graph of $y=f(x)$ within the lines are reversed in direction and spread over the plane outside the lines in the graph of $y=f(1/x)$.

IX. Given the graph of $y=f(x)$ to find the graph of $y=\sqrt{f(x)}\equiv F(x)$. (See Fig. 6, in which the graphs of $y=(x-1)(x-3)$ and $y=\sqrt{(x-1)(x-3)}$ are shown.)

Since the ordinates of the graph of $y=F(x)$ are the square roots of the corresponding ordinates of $y=f(x)$, the two curves meet on the x-axis and on the line $y=1$. Between the lines $y=0$ and $y=1$, the graph of $y=\sqrt{f(x)}$ lies above the graph of $y=f(x)$. Above the line $y=1$ the graph of $y=\sqrt{f(x)}$ is below the graph of $y=f(x)$. That is to say, the graph of $y=\sqrt{f(x)}$ is always between the graph of $y=f(x)$ and the line $y=1$.

If the graph of $y=f(x)$ passes below the x-axis the graph of $y=\sqrt{f(x)}$ becomes discontinuous. This leads to certain interesting results. If, for example, the term $\sqrt{-x^2}$ is added to any function the real graph of the function is thereby destroyed save for the points where it crosses the y-axis. Similarly $\sqrt{x}$ or $\sqrt{-x}$ when added to a function destroys the half of the graph to the left or right of the y-axis and transforms to a certain extent what remains. More generally the real graph of a function may be destroyed in the interval from a to b by adding the term $\sqrt{(x-a)(x-b)}$ or outside the interval by add-

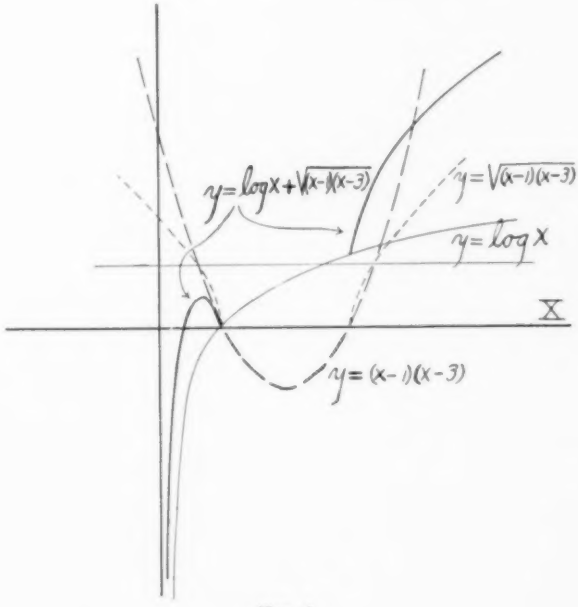


FIG. 6.

ing $\sqrt{(a-x)(x-b)}$. The graph is of course transformed to a certain extent where it is not destroyed since a term has been added. This transformation may however be reduced practically to nothing for regions ordinarily plotted by placing the radical over a large denominator, as for example, 1000^{1000}. As an illustration the graph of $y=\log x+\sqrt{(x-1)(x-3)}$ is given. (See Fig. 6.) The scheme of plotting will be clear from the figure. By introducing additional factors under the radical the graph may be destroyed throughout various intervals. It may also be destroyed in part or entire save for a set of isolated points by adding terms of the following type: $\sqrt{x}\sin nx$, $\sqrt{-x}\sin nx$, $\sqrt{-x^2}\sin nx$. As an example the graph of

$$y=\frac{x^2}{16}+\sqrt{-x^2}\sin 3x$$

is given. (See Fig. 7.) The graph consists simply of the isolated points marked by small circles.

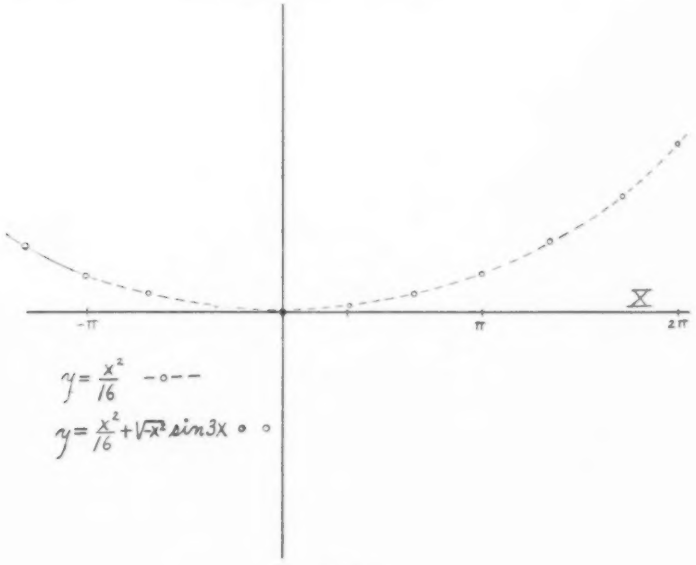


FIG. 7.

X. Given the graph of $y=f(x)$ to find the graph of $x=f(y)$. The required graph differs from the given only in that the

x and y are interchanged. If $y=f(x)$ is plotted the form of $x=f(y)$ can be discovered by rotating the paper 180 degrees about the x-axis, turning it in its own plane 90 degrees about the origin in the positive direction of rotation, and then looking through the paper toward the light. This method is useful in deriving the graphs of inverse functions.

In closing it may be noticed that irreducible rational integral functions, save those of the first, the second, and possibly the third degrees, are troublesome forms to plot by any method. The process of evaluating the function by synthetic division for a sufficient number of values of x to give a fair approximation to the curve is long and tiresome. The determination of maxima and minima is troublesome, since the degree of the first derivative is but one less than the degree of the original function. The application of certain of the methods described above may possibly shorten the amount of work in some cases. The two

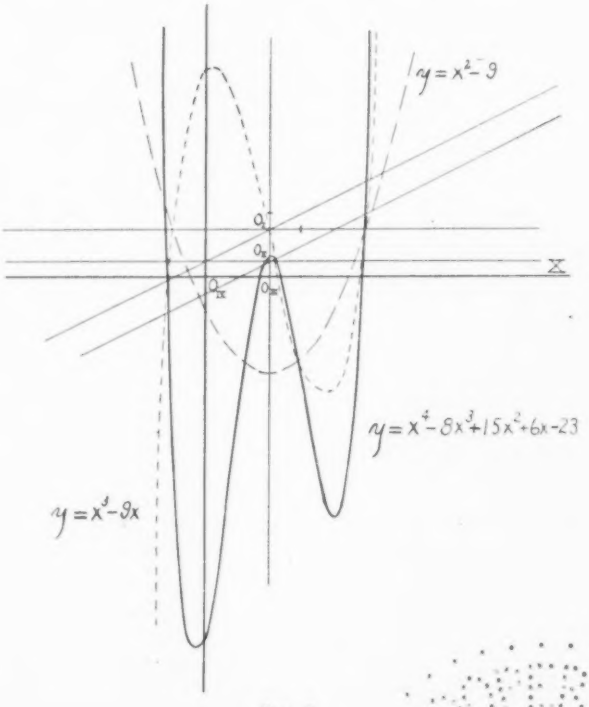


FIG. 8.

points of method that are most useful are, first, the movement of the x-axis to eliminate the absolute term and, second, the movement of the y-axis to eliminate any other term save that of highest degree. To determine the distance the y-axis must be moved in order to eliminate a certain term, $x + c$ is substituted for x, and the new function having been arranged in powers of x the coefficient of the term in question is equated to zero and the value of c determined. Fig. 8 gives the graph of

$$y = x^4 - 8x^3 + 15x^2 + 6x - 23,$$

the unit on the y-axis being but half that on the x. The scheme used in plotting this graph is given below and also a second optional scheme suggesting another method that may in some cases be helpful in plotting the graphs of irreducible rational integral functions.

SCHEME FOR PLOTTING $y = x^4 - 8x^3 + 15x^2 + 6x - 23$.

Step.	Result.
Plot $y = x^2 - 9$ and $y = x$ and multiply..	$y = x(x^2 - 9) = x^3 - 9x$
Lower x-axis two units	$y = x^3 - 9x + 2$
Plot $y = x$ and multiply graphs	$y = x^4 - 9x^2 + 2x$
Lower x-axis one unit	$y = x^4 - 9x^2 + 2x + 1$
Move y-axis two units to the left	$y = x^4 - 8x^3 + 15x^2 + 6x - 23$

OPTIONAL SCHEME FOR PLOTTING $y = x^4 - 8x^3 + 15x^2 + 6x - 23$.

Step.	Result.
Plot $y = x^4$.	
Move y-axis two units to left,	$y = f(x - 2) = x^4 - 8x^3 + 24x^2 - 32x + 16$
Plot $y = -9x^2 + 38x - 39$.	
Add the two latter graphs..	$y = x^4 - 8x^3 + 15x^2 + 6x - 23$

One additional illustration is given of the construction of a graph which to a student ignorant of methods of determining singular points might present considerable difficulty unless attacked by some of the methods described above. (See Fig. 9.)

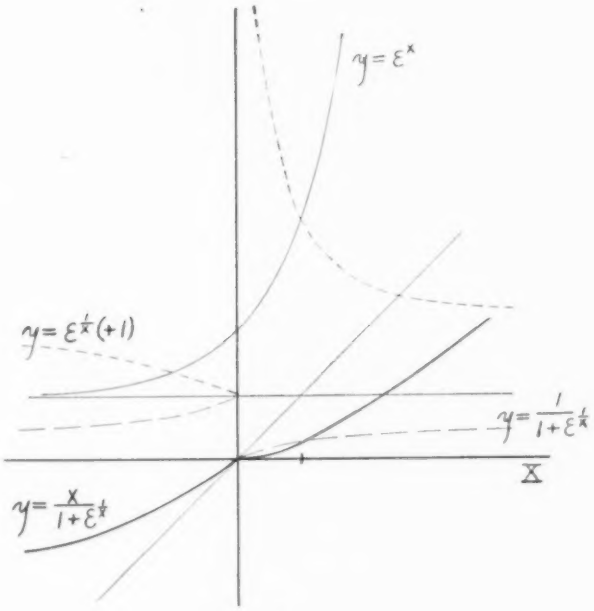


FIG. 9.

SCHEME OF PLOTTING $y=\frac{x}{1+e^{1/x}}$.

Step.	Result.
Plot $y=f(x)=e^x$.	
Plot $y=f\left(\frac{1}{x}\right)$	$y=e^{1/x}$
Lower x-axis one unit	$y=e^{1/x}+1$
Plot $y=\frac{1}{f(x)}$	$y=\frac{1}{1+e^{1/x}}$
Plot $y=x$ and multiply by above	$v=\frac{x}{1+e^{1/x}}$

BOSTON UNIVERSITY,
BOSTON, MASS.

ARE YOUR TEACHERS AIDS OR HINDRANCES TO SCHOOL PROGRESS?

(By the Teachers and Professional Service Division of the Employment Bureau of the United States Department of Labor.)

Wherever we go we hear the cry "Better Schools." Our legislatures have passed numerous laws, and our state departments of education have promulgated various regulations toward the improvement of our schools. The Federal Bureau of Education has contributed much in this direction. To any reasoning mind it must be apparent that "better schools" means primarily "better teachers." The National Education Association, in a resolution passed at the 1916 session (July, New York City), stated:

"The National Education Association calls the attention of the American people to the fact that teaching is a profession, demanding for its successful practice a technical training that will put the teacher in possession of professional standards; that these professional standards can be maintained only by the employment of superintendents, supervisors and teachers who have unquestioned professional qualifications for their work; that members of the teaching profession can have but one client, the public."

While our legislatures have enacted laws specifying the qualifications required in teachers, only in a few instances has provision been made for supplying these teachers. The result is that school officers are frequently at a loss to know where they shall secure the teachers with which to fill the vacancies occuring, and hence persons not properly qualified are frequently selected. Local politics are also a prominent factor in many instances. The following table, showing the degree of preparation on the part of teachers in various states, will prove of interest:

State.	Total Teacher.	Graduates of				Attended (Not Graduated).		
		Common School Only.	High School Only.	Normal School Only.	College or University.	H. S. 1–3 Years.	Normal School.	College or University.
Illinois (1914)	30,565	1,596	9,688	4,193	4,073	2,988	5,875	2,251
" (h. s.)	2,808		103	359	1,980	13	83	354
Kansas (rural)	7,721	1,231	4,791	225		1,474		
" (grades)	5,202	471	2,556	747	328		659	440
" (h. s.)	2,476	55	174	651	1,352			244
Kentucky (rural)	10,381	7,640					2,741*	
" (city)	2,071	1,172				371	192	336
Oregon	6,173	180	1,122	737	564	618	340	475
Wisconsin:								
Rural	6,630	118	3,062	2,245	30	581	791	38
State graded	1,665	16	368	504	31	66	318	35
Grade below h. s.	1,220		370	685	33	17	222	26
Rural h. s.	1,106		7	388†	487		59‡	139§
City elementary	4,047		185	2,910	149	79	345	186
City h. s.	1,349			135	885	21	19‖	78

The above table reveals the deplorable condition that in the five states mentioned, of 80,606 teachers, 12,479 have no further preparation than that gained in the common schools; that 3,480 are holding positions as grade teachers (not rural) who have no preparation above high school, and that 533 have attended but have not graduated from high school; that 55 high-school teachers have no higher preparation than that gained in the common schools, 34 have attended but have not graduated from high school, and 284 have graduated from no school above high school; that 1,533 high-school teachers have graduated from no school above the normal school, and 161 have attended but not graduated from normal school, a total of 18,559 teachers lacking adequate preparation for the positions they hold. In other words, of the total number of teachers in these states, 23 per cent. are not qualified for their work, and 15.5 per cent. have attended no school above the common school. Surely this is not as it should be. We are sending our children to the schools hoping that there they may gain a substantial foundation for

* Includes graduates.

† Includes 42 who have not attended high school and 42 attended but not graduates of high school.

‡ Includes 5 not graduates of high school.

§ Includes 93 normal-school graduates.

‖ Includes 15 who have not attended high school.

true citizenship and the necessary instruction to prepare them for their work in life. The teachers should, therefore, be the best the money available can procure.

To aid in this, to help school officers in locating competent teachers for their vacancies, the United States government has created, as a part of its employment service, the Teachers and Professional Service Division. The methods used in this division are most thorough, and when the government bureau recommends a teacher for a specific position, its judgment may be relied upon, for no teacher is recommended unless the bureau is fully assured of the candidate's unquestioned fitness for the position. Its standard of scholarship, while high, is not unreasonable:

Rural School Teachers—graduates of normal training high schools or better.

Graded School Teachers—graduates of recognized normal schools providing two or more years above high schools.

High School Teachers—graduates of recognized colleges or universities. The course of study should include at least sixteen hours of education.

The bureau has found its greatest obstacle to success in the salaries offered in some districts, where a minimum salary is offered and a maximum of scholarship is required. In the experience of the bureau it is impossible to find qualified teachers below the following salaries:

Rural Schools—$40 a month for inexperienced teachers.
$50 to $65 a month for experienced teachers, depending on experience.

Graded Schools—$65 a month for inexperienced teachers.
$70 to $85 a month for experienced teachers, depending on experience.

High Schools—$65 to $75 a month for inexperienced female teachers.
$70 to $80 a month for inexperienced male teachers.
$80 to $95 a month for experienced female teachers, depending on experience.
$85 to $100 a month for experienced male teachers, depending on experience.

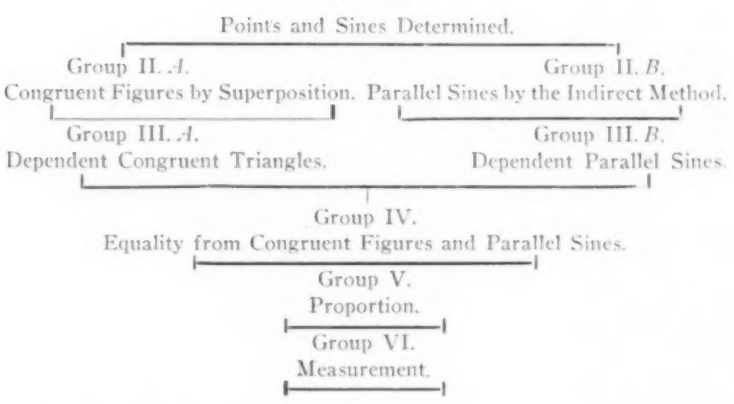
The above are offered as *minimum* salaries. It must be understood, however, that higher salaries mean better teachers. A good teacher will not accept a low salary, and any teacher of experience who accepts a salary less than those shown in the above schedule should be looked upon with suspicion. The observations of the bureau disclose abundant reason for such suspicion, and school officers will do well to bear this in mind when contending for low salaries.

Another difficulty, not so widespread as the foregoing, is the seeming inability of some graduates to properly estimate the value of their services. In some instances new college graduates have specified $100 a month as a minimum salary. While it is never the policy of the bureau to recommend a decrease in the salaries offered, it is sometimes unable to place beginners because they have placed their minimum salary too high. A beginner should remember that his or her chief consideration must be effecting an entry into the profession. After once entering upon the profession, salary should increase at the rate of $5 a month for each of the first four years. Further increases can only be had by locating in larger systems.

In conclusion allow me to say that it is the earnest desire of the Teachers and Professional Service Division to aid school officers and teachers to the fullest extent possible. We must, however, have the hearty co-operation of both—of the former in reporting vacancies and referring teachers for registration, and the latter in registering and keeping the office informed of all changes in location and availability, and responding promptly when an effort is made to place them in a position. The service is free to all. All communications should be addressed: Teachers and Professional Service Division, U. S. Employment Service, 845 South Wabash Avenue, Chicago, Illinois.

AN OUTLINE OF PLANE GEOMETRY AS USED IN THE DURFEE HIGH SCHOOL.

By Robert R. Goff.



In group I. are:

(*a*) Through a point only one straight line can be drawn making a given angle with a second straight line.

(*b*) Through two points only one straight line can be drawn.

(*c*) Through a point only one straight line can be drawn perpendicular to a second straight line.

(*d*) Through a point only one straight line can be drawn parallel to a second straight line.

(*e*) Two straight lines can intersect in only one point.

(*f*) Only one circle can be drawn with a given center and a given radius.

a needs a word of explanation. If one line crosses a second line, four angles are formed but *the angle* between the lines is that angle formed by the revolution of the first line, counter-clockwise, from the position of the second line. There are two cases in *a*:

1. Point on the line; 2. Point outside the line.

(b) is a special case of (a), 1 with a zero angle.

(c) is a special case of (a), 1 and 2.

(d) is a special case of (a), 2 with a zero angle.

Group I. is usually assumed and sometimes the two basic theorems of measurement, the measure of a central angle, and of a rectangle. In a few courses group II. A and B are also assumed.

In group II. A are:

(a) All straight angles are equal.

(b) If two triangles have two sides and the included angle of the one equal respectively to two sides and the included angle of the other, the triangles are congruent.

(c) Similar to (b), but with two angles and a side.

(d) If two parallelograms have two adjacent sides and the included equal, etc.

(e) A diameter of a circle bisects the circle.

(f) In the same or equal circles, equal central angles intercept equal arcs.

(g) Converse of f.

These theorems are proved by superposition and group I.

(c) includes "two angles and the included side" and can be proved by (a) in group I.

(f) may be worded: Two sectors are congruent if they have two straight sides and the included angle equal, etc.

(g) may be worded: Two sectors are congruent if they have three sides equal, etc.

In group II. B are:

(a) If two straight lines are cut by a transversal making the corresponding angles equal, the lines are parallel.

(b) Two straight lines parallel to a third straight line are parallel.

These are proved by the indirect method and group I.

(a) can be proved by (a) in group I.; (b) by (d).

In group III. the dependent cases of congruent triangles are the right triangles and the scalene triangles having three sides of the one equal to three sides of the other. They all depend upon (b) or (c) in group II. A.

The common theorems of parallel lines depend directly or indirectly upon (*a*) in group II. *B*.

Group IV. is equality derived from congruent figures or from parallel lines. The first theorem of equal angles from parallel lines can be either alternate-interior angles or corresponding angles. The proof is indirect, using (*d*) in group I.

In group V. is proved the last important theorem for parallel lines: If a line divides two sides of a triangle proportionally, it is parallel to the third side. The proof is indirect.

The four cases using the indirect proof, mentioned above, involve parallel lines and use the simplest elementary principles as group I. or the general axioms. In no other theorems is the indirect method favored except, perhaps, in one or two cases of inequality and here, even, it can usually be avoided.

A, 1 in group I. is not true in the geometry of space and is the basis of the plane theorems that are not necessarily true in space. The common theorems of this kind are:

(*a*) Group I., *c*.

(*b*) Two perpendiculars to the same straight line are parallel.

(*c*) A straight line perpendicular to a radius at its end is tangent to the circle.

DURFEE HIGH SCHOOL,
FALL RIVER, MASS.

NEW BOOKS.

Differential Equations. By EDWARD J. MAURUS. New York: Ginn and Company. Pp. 51. 72 cents.

This is an elementary course and covers differential equations of the first order and first degree, of the first order and higher degrees, of orders higher than the first. Some applications are given at the end.

The Theory of Maxima and Minima. By HARRIS HANCOCK. Boston: Ginn and Company. Pp. 193. $2.50.

Professor Hancock has done English-speaking students of mathematics a great service in writing this volume on Maxima and Minima. It treats of ordinary cases "where the functions are everywhere regular and where the forms are either definite or indefinite" and also of the special cases "where only one-sided differentiation enters the 'ambiguous case' where the form is semi-definite, etc." There is given a chapter on the Scheeffer theory, one where subsidiary conditions connect the variables, and one on fundamental conceptions in the theory of analytical functions. The treatment of all topics is careful and clear.

Inorganic Chemistry. By HORACE G. BYERS. New York: Charles Scribner's Sons. Pp. 651.

This book has been written to meet the needs of those who have had chemistry in high school as well as for those who have not; for those who are to use it as a tool as well as for those taking it as a culture course; and is designed to require a year's time of three hours a week for those who have had high-school chemistry and a year and a half for those who have not.

It seems to be a very well-written and teachable book.

Record Forms for Vocational Schools. By JOSEPH J. EATON. Vol. VI of the School Efficiency Monographs. Yonkers-on-Hudson, N. Y.: The World Book Co. Pp. vii + 56. Price 60 cents.

Every vocational school and every executive responsible for one should have a copy of this book. It shows how to apply the methods of business accounting to the work of those schools that combine a business with teaching. Much labor and confusion can be saved by such methods and forms as it advocates.

The Science and the Art of Teaching. By DANIEL WOLFORD LA RUE. New York: American Book Co. Pp. 336.

The author's courses in teaching have built up the subject matter of

this book. It divides into Method as Determined by the Nature of the Child, as Related to the Teacher, as Conditioned by Subject Matter, and as Influenced by the Educational Ideals.

The work is well adapted to training school or normal school pupils who are preparing to teach, but it should, of course, be supplemented by much other reading. A good bibliography is included.

The best feature of the book is its simplicity and the logical way in which it develops its topics. There is considerable difference in the quality of different parts of the work, and, as in most such texts, some of the things expressed as facts are open to question.

Polyanna Annual, the Yearly Glad Book. Edited by FLORENCE ORVILLE. Boston: The Page Co. Pp. 246. Price $1.35.

This is a collection of stories, poetry, and suggestions of interesting things for children to do. The material is well chosen and attractively arranged, with plenty of illustrations, some of which are in color. It should prove both interesting and useful to the children.

Experimental General Science. By WILLARD NELSON CLUTE. Philadelphia: P. Blakiston's Son & Co. Pp. xv + 303. Price $1.25.

This is an interesting and practical high school book on the foundations of science. It prepares the children for later sciences, and gives them a clear idea of the investigations, simple enough to be within their range. The book includes a discussion of the science of the human body.

Rural Arithmetic. By AUGUSTUS O. THOMAS. New York: American Book Co. Pp. 288.

The author has succeeded very well in using the material that is likely to be familiar to the country child. There is an abundance of interesting subjects, which are not only giving practice in arithmetic, but also are broadening the child's outlook.

Much of the book could be used to advantage as an auxiliary text, even in city schools.

Our Little Frankish Cousin of Long Ago. By EVALEEN STEIN. Boston: The Page Co. Pp. 116. Price 60 cents.

This is the latest addition to "The Little Cousin" series, and conforms to the others in its makeup. It tells, in a very interesting way, the story of a boy at the Court of Charlemagne, and so gives an excellent picture of the life of this period. It should be valuable as a supplementary reader for school libraries.

Science for Beginners. By DELOS FALL. Yonkers-on-Hudson, N. Y.: World Book Co. Pp. 382.

This is a general science for use in grammar schools and junior high schools. It is an unusually practical book in that it explains the "sci-

entific method" very clearly, and then leads the pupil into simple investigations of natural phenomena. There can be no doubt of the interest pupils would take in a course founded on this book.

Our Little Roumanian Cousin. By CLARA VOSTROVSKY WINLOW. Boston: The Page Co. Pp. 113. Price 60 cents.

This book of "The Little Cousin" series is particularly timely now, when everyone is more than ever interested in the Balkan States. It tells of the customs of the Roumanians in an interesting and simple way.

Standard for the Evaluation of Efficiency in Palmer Method Handwriting. By A. N. PALMER. New York: The A. N. Palmer Co., 30 Irving Place.

The author of the Palmer method of writing has devised a set of eight standards, one for each of the elementary school grades, for the purpose of measuring penmanship in position, movement, speed and form.

With the standards is a tabulating pad that reduces greatly the time needed for grading a class.

This set promises to be a valuable contribution to the material on handwriting.

Problems in the Mathematical Theory of Investment. By GUY ROGER CLEMENTS. Boston: Ginn and Co. Pp. 24. Price 32 cents.

This is a list of one hundred problems to supplement and extend books on the theory of investment. The introduction states the basic formulas and outlines the method of attack.

The problems seem well chosen, both in range and difficulty.

Hamilton's Standard Arithmetics. By SAMUEL HAMILTON. Book I., for the first four grades, pp. 256, price 44 cents; Book II., for fifth and sixth grades, pp. 300, price 48 cents; Book III., for seventh and eighth grades, pp. 374, price 56 cents. New York: The American Book Co.

This series follows earlier books by the same author. The books gain by their elimination of some topics that have been agreed upon as unnecessary, and by the introduction of much new material that is interesting and practical. Among the useful parts are chapters on household problems, shop problems, and problems of business, farm and factory.

Commercial Algebra, Book I. By GEORGE WENTWORTH, DAVID EUGENE SMITH, and WILLIAM S. SCHLAUCH. Boston: Ginn and Co. Pp. 266. Price $1.12.

This book in intended for the first course in algebra for commercial high schools. It retains the usual topics only in so far as they have commercial bearing. The authors believe, however, that such a general foundation has been laid that another half year's work will prepare a student for college.

The publication of the book follows an exhaustive study of commercial mathematics, made by Mr. Schlauch, and this fact guarantees its value in its field.

Book II. is in preparation.

Numerical Trigonometry. By N. J. CHIGNELL. Oxford: The Clarendon Press. Pp. xii + 127.

This is a most interesting little book, in which the first five chapters are given up to the solution of triangles by means of ratios and right triangles. The sixth and seventh chapters take up algebraic methods and additive and projective formulas.

The idea of the text is to make possible the early introduction of practical trigonometry.

Plane Geometry, Revised Edition. By H. E. SLAUGHT and N. J. LENNES. Boston: Allyn and Bacon. Pp. 310. Price $1.00.

In this edition the authors retain the main features of their former book, the human interest side being represented by historical notes, diagrams of applied geometry, etc.

The number of theorems has been somewhat reduced, and the book has a more attractive interior.

Descriptive Geometry. By JOHN C. TRACY and HERBERT B. NORTH. New York: John Wiley and Sons. Pp. x + 126. Price $2.00.

In this text the shape—it is 7½ inches high and 10½ inches wide—gives opportunity for some unusually well-planned pages. They not only have the figures, which are many and excellent, on the pages with the text concerning them, but they also present a very clear and appealing makeup.

The subject matter is well chosen and carefully written.

NOTES AND NEWS.

THE Crop Improvement Committee, 65 Board of Trade, Chicago, will send any teacher full directions for testing seed corn. The committee is conducting a campaign, urging the schools to undertake the seed testing for the farms, in this way helping to improve our crops. More than 75,000 children helped in this work last year, and the committee hopes to increase this number very greatly this year.

THE Southern Section of the Association of Teachers of Mathematics in the Middle States and Maryland held its first meeting of the year on Saturday, January 26, 1918, at Goucher College, Baltimore. The morning addresses were: "Mathematics as a Preparation for Technical Education," by Henry A. Converse, Polytechnic Institute, Baltimore, and "The Advantages of Mathematics for Girls," by Alice Deal, McKinley Manual Training School, Washington, D. C.

MR. UPTON, of Teachers College, who was to have addressed the afternoon meeting, was unfortunately prevented from attending; but Mr. Wm. E. Breckenridge, of Teachers College, kindly delivered for him the admirable address, illustrated with the stereopticon, on "High School Mathematics as an Aid to the Interpretation of Life about Us." Although the attendance at the meetings was small, the enthusiasm of those present made the animated discussions which followed the addresses peculiarly instructive. Officers of the preceding year were reelected.

THE spring meeting of the Southern Section was held at the Central High School, Washington, on April 28. The morning session was given up to an "Experience Meeting," the speakers being Howard L. Hodgkins, Florence P. Lewis, Miss Lipschutz, Henry A. Converse, Caroline Becker, Cecil A. Ewing, Edna Leola Dixon, George A. Ross, Elizabeth White, Mary Draney, Janet McWilliams, Alice Deal. In the afternoon William H. Wetzel, principal of the high school, Trenton, N. J., spoke on "The Junior High School and its Influence on the

Curriculum in Mathematics." The discussion was opened by Cecil A. Ewing, of Tome School.

THE Johns Hopkins University announces a series of "Studies in Education." "The Correlation of Abilities in High School Pupils," by David E. Weglein, and "Experimental Study of Motor Abilities of Children in Primary Grades," by Buford Jennette Johnson, are already in print.

THE John C. Winston Company, of Philadelphia, announced its entrance into the school-book field. Among its first books will be a series of readers, several books on civics, and a set of grade histories.

A Christmas Suggestion.—When you make a present of *The Youth's Companion* you are giving not merely the means of wholesome pleasure and fascinating information every week. *The Companion* is all that. But it is something more. Hundreds of letters to *The Companion* speak of the influence of the paper in binding home ties. The mothers and fathers and the boys and girls in *Companion* families are very close knit in their affections. They have a common interest in the same duties and recreations, and they all agreed *The Companion* as one of themselves. It has a personality and a character unique among publications, and you cannot introduce a more inspiring influence into any home circle.

It is not a publication merely—it's a *friend*. *The Companion* alone is $2.00, but the publishers make an *extraordinary double Christmas present offer.—The Youth's Companion* and *McCall's Magazine* together for $2.25.

This two-at-one-price offer includes: (1) *The Youth's Companion*—52 issues in 1918; (2) all remaining 1917 issues of *The Companion* free; (3) *The Companion* Home Calendar for 1918; (4) *McCall's Magazine*—12 fashion numbers in 1918; all for only $2.25. (*The Youth's Companion,* Commonwealth Ave., Boston, Mass.)

THE regular fall meeting of the Philadelphia section of the Association of Teachers of Mathematics in the Middle States and Maryland was held at the Central High School on Wednesday, November 7, 1917. Dr. Rorer, of the William Penn High School, gave a talk on "General Mathematics for the First

Year High School," and Professor A. H. Wilson, of Haverford College, read a paper on "General Mathematics for College." There followed a very interesting discussion of these papers.

There has been no mid-winter meeting of this section, for it was decided that, owing to war conditions, it would be better to omit the usual banquet.

THE Mathematics Club of the University of North Carolina held an informal smoker on the night of Wednesday, February 6. A paper on "A Method for Finding the Complex Roots of a Cubic Equation" was presented to the club by Mr. J. W. Lasley, Jr., of the department of mathematics. A noteworthy feature of this meeting of the club was the presence of certain of the best men in the freshman class. To interest these men at the outset of their college career and to get into a closer touch with them than the class room affords are among the aims of the organization.

DR. DAVID EUGENE SMITH, of Columbia, addressed the Association of Teachers of Secondary Mathematics of North Carolina in Greensboro, February 1 and 2. A popular lecture on "The Origin of Mathematics," a somewhat more technical one on "Deficiencies in Present Preparatory Mathematics" and a round-table discussion on the topic "A Proper Approach to Elementary Mathematics" were the contributions of Dr. Smith. Papers were presented to the association by Miss Fannie Starr Mitchell, of the Raleigh High School, Mr. J. W. Lasley, Jr., of the University of North Carolina, Miss Virginia Ragsdale, of the Normal College, and Miss Ella Bradley, of the Gastonia High School. The following officers were elected: Mr. W. W. Rankin, Jr., of the University of North Carolina, president; Mr. S. L. Sheep, of the Marion High School, first vice-president; Miss Maria Graham, of the Eastern Carolina Training School, second vice-president; Miss Nita Gressitt, of the Greensboro High School, third vice-president; Mr. J. W. Lasley, Jr., secretary.

NEW MEMBERS.

Frederick Albert Lewis, Boys' High School, Frederick, Md.
Ernest H. Nicholls, Liberty, N. Y.
Howard F. Knapp, Genoa, N. Y.
Nellie C. Walrad, White Plains, N. Y.
Beatrice Reynolds, Galiton, Pa.
Charlotte E. Babcock, Leonardsville, N. Y.
Carrie Harbaugh, Middleburg, Md.
Joana Van Daniker, 805 N. Fremont Ave., Baltimore, Md.
Mary W. Carroll, Chestertown, Md.
Helen M. Hibbard, 701 College Ave., Elmira, N. Y.
Mary E. Caster, 448 Van Houten St., Paterson, N. J.
Raleigh Schoiling, The Lincoln School, New York City.
Clarence E. Rushmer, High School, Binghamton, N. Y.
M. G. Schucker, 1306 Lancaster Ave., Pittsburgh, Pa.
A. B. McClure, South Hills High School, Pittsburgh, Pa.
N. May Stiles, 286 Aycrigg Ave., Passaic, N. J.
John J. Greer, 11 E. 36th St., New York City.
Mary Ellen Dwyer, 1630 E. Oliver St., Baltimore, Md.
Elizabeth Malorly, School 94, Chase & McDonough Sts., Baltimore, Md.
Anna A. Silvernail, 115 First St., Scotia, N. Y.
Alta Blanche Chase, 423 W. 120th St., New York City.
A. A. McCroue, Aldan, Pa.